



## Horizontal Curvature Friction in Post-Tensioning

The effect of horizontal curvature is to be considered in the design of prestressed concrete structures. It is a portion of the friction loss due to the angular change of the prestressing steel. Friction losses are calculated using the formula,

$$T_o = T_x e^{(KL + \mu\alpha)}$$

then,

$$T_x = T_o e^{-(KL + \mu\alpha)}$$

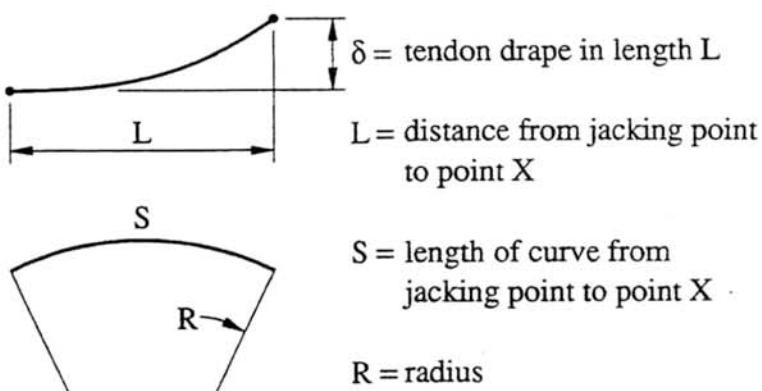
See *Bridge Design Specifications*, Article 9.16.1.

$\alpha$  = Total angular change of prestressing profile in radians from jacking end to Point X. Where  $\alpha$  is the vectorial summation of the tendon drape and the horizontal curvature.

$$\alpha = \sqrt{(\alpha_v)^2 + (\alpha_h)^2}$$

$$\alpha_v = \frac{2\delta}{L}$$

$$\alpha_h = S/R$$



The BDS program has been modified to take into account the horizontal curvature. Attached is a example of the input sheet.

The following example calculations demonstrate how the program computes the effect of horizontal curvature.

Jerry A. McKee

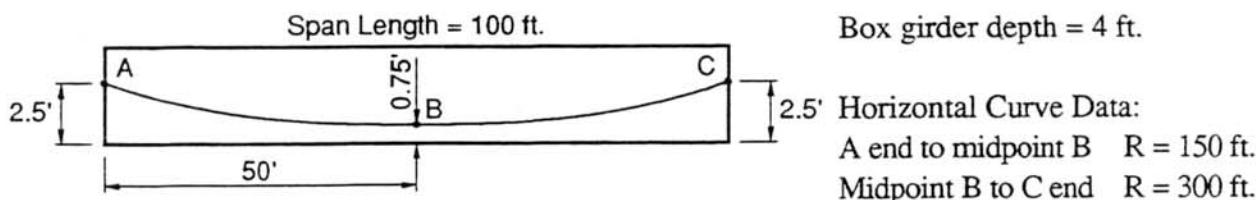
JWO:jgf

Floyd L. Mellon

*Supersedes Memo to Designers 11-30 dated April 1990*

Long Hand Example: refer to *Bridge Design Specifications*, Article 9.16.1 for terms.

### Typical Box Girder



Solve for  $T_x$  at B:

From A → B

$$\alpha_v = \frac{2\delta}{L} = \frac{2(2.5 - 0.75)}{50} = 0.07 \text{ Radians}$$

$$\alpha_h = \frac{S}{R} = \frac{50}{150} = 0.33 \text{ Radians}$$

$$\alpha = \sqrt{(\alpha_v)^2 + (\alpha_h)^2} = \sqrt{(0.07)^2 + (0.33)^2} = 0.34 \text{ Radians}$$

$$T_x = T_o e^{-(KL + \mu\alpha)}$$

$T_o = 0.75 f'_s$  for low relaxation strand (*Bridge Design Specifications*, Article 9.15.1)

$$T_o = 0.75 (270) = 202.5 \text{ ksi}$$

$$\mu = 0.20$$

$$K = 0.0$$

$$T_x = T_o e^{-(0.20)(0.34)}$$

$$= 202.5 e^{-(0.20)(0.34)}$$

$$= 189.2 \text{ ksi or } 0.93 P_j$$

Solve for  $T_x$  at C:

From B → C

$$\alpha_v = \frac{2(2.5 - 0.75)}{50} = 0.07 \text{ Radians}$$

$$\alpha_h = \frac{50}{300} = 0.17 \text{ Radians}$$

$$\text{Total } \alpha_v = 0.07 + 0.07 = 0.14 \text{ Radians}$$

$$\text{Total } \alpha_h = 0.33 + 0.17 = 0.50 \text{ Radians}$$

$$\alpha = \sqrt{(0.14)^2 + (0.50)^2} = 0.52 \text{ Radians}$$

$$T_x = 202.5 e^{-(0.20)(0.52)}$$

$$= 182.5 \text{ ksi or } 0.90 P_j$$



**BRIDGE DESIGN SYSTEM**  
Horizontal Curvature Data

ACCOUNT
1 2 3 4 5 6 7 8

DISTANCE FROM BB TO BC	CURVE NO. 1			CURVE NO. 2			CURVE NO. 3			CURVE NO. 4			CURVE NO. 5		
	RADIUS (F.T.)	LENGTH (F.T.)													
9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77															

## NOTES:

1. The maximum number of curves, including tangents, allowable for input is limited to five (5).
2. The distance from BB to BC may be NEGATIVE, ZERO or POSITIVE. If the beginning of the bridge is on a curve, then the distance from BB to BC must be NEGATIVE.
3. No adjustment on the wobble factors is required.
4. Leave RADIUS input field blank for tangents.
5. Input data will be used to compute the horizontal angle of a given cable path.
6. Round the radius to the nearest tenth of a foot and the length to the nearest hundredth of a foot.
7. Curvature data to be entered in sequence along the structure layout line or the centerline of the structure.

## 11-30 HORIZONTAL CURVATURE FRICTION IN POST-TENSIONING

The effect of horizontal curvature is to be considered in the design of prestressed concrete structures. It is a portion of the friction loss due to the angular change of the prestressing steel. Friction losses are calculated using the formula,

$$T_0 = T_X e^{(KL + \mu\alpha)}$$

then,

$$T_X = T_0 e^{-(KL + \mu\alpha)}$$

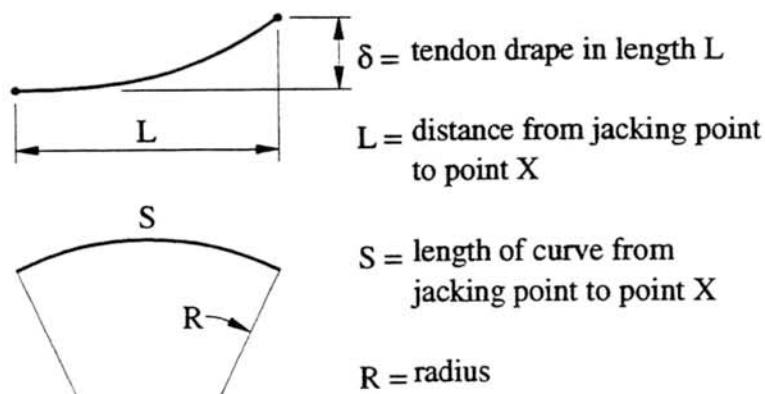
See *Bridge Design Specifications*, Article 9.16.1.

$\alpha$  = Total angular change of prestressing profile in radians from jacking end to Point X. Where  $\alpha$  is the vectorial summation of the tendon drape and the horizontal curvature.

$$\alpha = \sqrt{(\alpha_V)^2 + (\alpha_H)^2}$$

$$\alpha_V = \frac{2\delta}{L}$$

$$\alpha_H = S/R$$



The BDS program has been modified to take into account the horizontal curvature. Attached is a example of the input sheet.

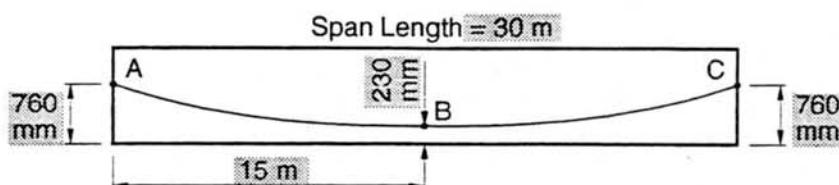
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Memo converted to metric.

The following example calculations demonstrate how the program computes the effect of horizontal curvature.

Long Hand Example: refer to *Bridge Design Specifications*, Article 9.16.1 for terms.

### Typical Box Girder



Box girder depth = 1300 mm

Horizontal Curve Data:

A end to midpoint B      R = 50 m

Midpoint B to C end      R = 100 m

Solve for  $T_x$  at B:

From A → B

$$\alpha_v = \frac{2\delta}{L} = \frac{2(760 - 230)1/1000}{15} = 0.07 \text{ Radians}$$

$$\alpha_h = \frac{s}{R} = \frac{15}{50} = 0.30 \text{ Radians}$$

$$\alpha = \sqrt{(\alpha_v)^2 + (\alpha_h)^2} = \sqrt{(0.07)^2 + (0.30)^2} = 0.31 \text{ Radians}$$

$$T_x = T_0 e^{-(KL + \mu\alpha)}$$

$T_0$  = 0.75 f' for low relaxation strand (*Bridge Design Specifications*, Article 9.15.1)

$$T_0 = 0.75 (1860) = 1395 \text{ MPa}$$

$$\mu = 0.20$$

$$K = 0.0$$

$$T_x = T_0 e^{-(0.20)(0.31)}$$

$$= 1395 e^{-(0.20)(0.31)}$$

$$= 1311 \text{ MPa or } 0.94 P_j$$

Solve for  $T_x$  at C:

From B → C

$$\alpha_v = \frac{2(760 - 230)1/1000}{15} = 0.07 \text{ Radians}$$

$$\alpha_h = \frac{15}{100} = 0.15 \text{ Radians}$$

$$\text{Total } \alpha_v = 0.07 + 0.07 = 0.14 \text{ Radians}$$

$$\text{Total } \alpha_h = 0.31 + 0.15 = 0.46 \text{ Radians}$$

$$\alpha = \sqrt{(0.14)^2 + (0.46)^2} = 0.48 \text{ Radians}$$

$$T_x = 1395 e^{-(0.20)(0.48)}$$

$$= 1268 \text{ MPa or } 0.91 P_j$$

Richard D. Land

Shannon H. Post

EKT/FH:jlw

**BRIDGE DESIGN SYSTEM**  
Horizontal Curvature Data

ACCOUNT	
1 2 3 4 5 6 7 8	

DISTANCE FROM BB TO BC	CURVE NO. 1		CURVE NO. 2		CURVE NO. 3		CURVE NO. 4		CURVE NO. 5	
	RADIUS	LENGTH								
(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)
0 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77										

(Card T)

NOTES:

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